

DFTT 24/96
May 1996

A CONFINEMENT MODEL CALCULATION OF $h_1(x)$

V. Barone^{a,1}, T. Calarco^b and A. Drago^b

^a*Dipartimento di Fisica Teorica, Università di Torino
and INFN, Sezione di Torino, 10125 Torino, Italy*

^b*Dipartimento di Fisica, Università di Ferrara
and INFN, Sezione di Ferrara, 44100 Ferrara, Italy*

Abstract

The transverse polarization distribution of quarks $h_1(x)$ is computed in a confinement model, the chiral chromodielectric model. The flavor structure of h_1 , its Q^2 evolution and Soffer's inequality are studied. The Drell-Yan double transverse asymmetry A_{TT} is evaluated and found to be one order of magnitude smaller than the double longitudinal asymmetry.

¹Also at II Facoltà di Scienze MFN, 15100 Alessandria, Italy.

The interest in the transverse polarization distribution of quarks and antiquarks, customarily called $h_1(x)$, has been recently strengthened by the perspective of its possible measurement in future collider experiments. Originally introduced by Ralston and Soper [1], who called it $h_T(x)$, $h_1(x)$ has been studied in detail, from a formal point of view, in more recent papers [2, 3]. The possible ways of measuring $h_1(x)$ in various hard processes have been also thoroughly investigated [3, 4, 5, 6]. Despite this intensive work, not much is actually known about the shape and the magnitude of $h_1(x)$. This is of course not surprising since h_1 , like all quark distributions, cannot be derived from the fundamental theory of strong interactions, QCD. At present we possess only: *i*) an admittedly crude evaluation of $h_1(x)$ in the simplest version of the MIT bag model [2], and *ii*) an estimate of its first moment – the so-called tensor charge – obtained by QCD sum rule methods [7]. It is clear that a more sophisticated model calculation of $h_1(x)$ is called for. This would also provide useful indications about the concrete possibility of a measurement of $h_1(x)$ in the experiments which are now being planned.

The difficulty of an experimental determination of $h_1(x)$, which is a leading twist quantity, resides mainly in the fact that, being a chirally-odd distribution, it is not measurable in polarized deep inelastic scattering. The best method to extract $h_1(x)$ seems to be [4, 5] the Drell–Yan dilepton production with two transversely polarized proton beams, an experiment which will be performed in the near future at RHIC [8]. The Drell–Yan double transverse asymmetry A_{TT} contains information on the flavor structure of h_1 . Therefore it would be important to predict the magnitude of A_{TT} in order to test the feasibility of the experiment. The available model calculations of A_{TT} [5, 9] are all based on the assumption that h_1^q is approximately equal to the helicity distribution Δq . Although this is probably true at very low momentum scales, such as those at which confinement model computations are implicitly performed ($Q^2 \lesssim 1$ GeV 2 , see below), it is certainly not true at experimental Q^2 scales (say $Q^2 \geq 10$ GeV 2). The reason is that Δq and h_1^q evolve differently in Q^2 . Whereas the first moment of Δq is constant, the first moment of h_1^q decreases with increasing Q^2 , and the evolution in the x -shape is even more dramatically different. Again, a more firmly based evaluation

of A_{TT} is needed to check whether this quantity is comparable in magnitude with the double longitudinal asymmetry A_{LL} , as it is claimed in [5, 9].

Another issue which is certainly worth exploring in the framework of confinement models is the inequality among leading twist polarized and unpolarized distribution functions recently derived by Soffer [10] (see also [11]) and the possibility of its saturation.

In the following we shall provide a theoretical determination of $h_1(x)$ in a confinement model, the chiral chromodielectric model [12], which has been already successfully used to compute other leading twist structure functions and various nucleon properties. In particular, the flavor structure of h_1 will be described in detail and the effects of the peculiar QCD evolution of h_1 investigated. A prediction for A_{TT} will also be presented. As we shall see, due to the different evolution of h_1^q and Δq , A_{TT} turns out to be much smaller than A_{LL} .

The quark transverse polarization distribution reads [2]

$$h_1(x) = \frac{\sqrt{2}}{4\pi} \int d\xi^- e^{-ixp^+\xi^-} \langle NS_\perp | \psi_+^\dagger(\xi) \gamma_\perp \gamma_5 \psi_+(0) | NS_\perp \rangle|_{\xi^+=\xi_\perp=0}. \quad (1)$$

A similar expression holds for the antiquark distribution, with the exchange of ψ and ψ^\dagger . Note that in eq. (1) only the ‘good’ light-cone components of the fields, $\psi_+ = \frac{1}{2}\gamma_- \gamma_+ \psi$, appear, signaling that h_1 is a leading twist quantity. The h_1 distribution measures the difference in the number of quarks with transverse polarization parallel (\uparrow) and anti-parallel (\downarrow) to the proton transverse polarization. This can be made transparent by introducing the Pauli–Lubanski projectors $P_\perp^{\uparrow\downarrow} = \frac{1}{2}(1 \pm \gamma_\perp \gamma_5)$ and inserting a complete set of states $\{|X\rangle\}$ in eq. (1). We then get

$$h_1(x) = \frac{1}{\sqrt{2}} \sum_X \{ |\langle NS_\perp | P_\perp^\uparrow \psi_+(0) | X \rangle|^2 - |\langle NS_\perp | P_\perp^\downarrow \psi_+(0) | X \rangle|^2 \} \delta[(1-x)p^+ - p_X^+]. \quad (2)$$

In a (projected) mean-field approximation, the matrix elements in eq. (2) can be rewritten in terms of single-particle (quark or antiquark) matrix elements. For a flavor f one thus gets

$$\begin{aligned} h_1^f(x) &= \frac{1}{\sqrt{2}} \sum_\alpha \sum_m P(f, \alpha, m) \int \frac{dp_\alpha}{(2\pi)^3 (2p_\alpha^0)} A_\alpha(p_\alpha) \delta[(1-x)p^+ - p_\alpha^+] \\ &\quad \times \bar{\varphi}(p_\alpha, m) \gamma_+ \gamma_\perp \gamma_5 \varphi(p_\alpha, m), \end{aligned} \quad (3)$$

where φ is the single-quark wave function, m is the projection of the quark spin along the direction of the nucleon's spin, $P(f, \alpha, m)$ is the probability of extracting a quark of flavor f and spin m leaving a state generically labelled by the quantum number α . The overlap function $A_\alpha(p_\alpha)$ contains the details of the intermediate states and of the projection used to obtain a nucleon with definite linear momentum from a three-quark bag (see for instance [13, 14]). The intermediate states which contribute to eqs. (2,3) are $2q$ and $3q1\bar{q}$ states for the quark distribution, and $4q$ states for the antiquark distribution.

At this point, we can already discuss qualitatively the Soffer inequality [10], which reads

$$q^f(x) + \Delta q^f(x) \geq 2|h_1^f(x)|, \quad (4)$$

where Δq^f is the helicity distribution function and q^f the unpolarized density. This relation has been proved in the parton model [10, 11] (for a QCD-improved parton model discussion of Soffer's inequality see [15]), and is satisfied flavor by flavor by both the quark and the antiquark distributions. An interesting issue is whether Soffer's inequality is saturated in some quark model (which means that $|h_1^q|$ takes its maximal value). To clarify this problem let us write the various leading twist distributions in an explicit form

$$q^f(x) = \sum_{\alpha} \sum_m P(f, \alpha, m) F_{\alpha}(x) \quad (5)$$

$$\Delta q^f(x) = \sum_{\alpha} \sum_m P(f, \alpha, m) (-1)^{(m+3/2)} G_{\alpha}(x) \quad (6)$$

$$h_1^f(x) = \sum_{\alpha} \sum_m P(f, \alpha, m) (-1)^{(m+3/2)} H_{\alpha}(x), \quad (7)$$

where

$$\left. \begin{aligned} F_{\alpha}(x) \\ G_{\alpha}(x) \\ H_{\alpha}(x) \end{aligned} \right\} = \int \frac{d\mathbf{p}_{\alpha}}{(2\pi)^3(2p_{\alpha}^0)} A_{\alpha}(p_{\alpha}) \delta[(1-x)p^+ - p_{\alpha}^+] \times \frac{1}{2} \begin{cases} u^2(p_{\alpha}) + 2u(p_{\alpha})v(p_{\alpha})\frac{p_{\alpha}^z}{|\mathbf{p}_{\alpha}|} + v^2(p_{\alpha}) \\ u^2(p_{\alpha}) + 2u(p_{\alpha})v(p_{\alpha})\frac{p_{\alpha}^z}{|\mathbf{p}_{\alpha}|} + v^2(p_{\alpha})(2\left(\frac{p_{\alpha}^z}{|\mathbf{p}_{\alpha}|}\right)^2 - 1) \\ u^2(p_{\alpha}) + 2u(p_{\alpha})v(p_{\alpha})\frac{p_{\alpha}^z}{|\mathbf{p}_{\alpha}|} + v^2(p_{\alpha})(1 - \left(\frac{p_{\alpha}^{\perp}}{|\mathbf{p}_{\alpha}|}\right)^2) \end{cases}. \quad (8)$$

In eq. (8) p^\perp is the projection of the momentum in the plane perpendicular to the proton's trajectory (chosen to be the z axis), and the currents have been written in terms of the single quark wave-function in momentum space

$$\varphi(p, m) = \begin{pmatrix} u(p) \\ \boldsymbol{\sigma} \cdot \hat{\mathbf{p}} v(p) \end{pmatrix} \chi_m. \quad (9)$$

Notice that the three quantities $F_\alpha, G_\alpha, H_\alpha$ satisfy the equality: $F_\alpha(x) + G_\alpha(x) = 2H_\alpha(x)$. This has led to the erroneous conclusion [10] that the inequality (4) is saturated for a relativistic quark model, such as the MIT bag model. It is clear from eqs. (5-7) that the spin-flavor structure of the proton, which results in the appearance of the probabilities $P(f, \alpha, m)$, spoils this argument and prevents in general the saturation of the inequality.

Soffer's inequality is saturated only in very specific (and somehow unrealistic) cases. For instance, it is saturated when $P(f, \alpha, -1/2) = 0$, which happens if the proton is modeled as a bound state of a scalar diquark and a u quark. Of course, this is too a rough picture of the proton. However, it is interesting to note that in $SU(6)$ the Λ is a bound state of a scalar-isoscalar ud diquark and an s quark: the h_1 distribution of the latter then attains the maximal value compatible with (4).

Another instance of saturation is when $F_\alpha = G_\alpha = H_\alpha$ and $P(f, \alpha, -1/2) = 2P(f, \alpha, 1/2)$. It is easy to verify that this happens for the d quark distribution in a nonrelativistic model of the proton with an $SU(6)$ wavefunction.

Apart from the two particular cases illustrated above, Soffer's inequality should not be expected to be saturated, and indeed it is satisfied but not saturated in the model we present here.

The model of the nucleon that we use to compute all ingredients appearing in eq. (3) and then to evaluate the quark distribution functions is the chiral chromodielectric model (CCDM) [12]. The Lagrangian of the CCDM reads

$$\begin{aligned} \mathcal{L} = & i\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{g}{\chi}\bar{\psi}(\sigma + i\gamma_5\boldsymbol{\tau} \cdot \boldsymbol{\pi})\psi \\ & + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}M^2\chi^2 + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\boldsymbol{\pi})^2 - U(\sigma, \boldsymbol{\pi}), \end{aligned} \quad (10)$$

where $U(\sigma, \boldsymbol{\pi})$ is the usual mexican-hat potential, see *e.g.* [16]. \mathcal{L} describes a system of interacting quarks, pions, sigmas and a scalar-isoscalar chiral singlet field χ . The

parameters of the model are: the chiral meson masses $m_\pi = 0.14$ GeV, $m_\sigma = 1.2$ GeV, the pion decay constant $f_\pi = 93$ MeV, the quark–meson coupling constant g , and the mass M of the χ field. The parameters g and M , which are the only free parameters of the model, have been univoquely fixed by reproducing the average nucleon-delta mass and the isoscalar radius of the proton.

The CCDM Lagrangian (10) contains a single–minimum potential for the chromodielectric field χ : $V(\chi) = \frac{1}{2}M^2\chi^2$. A double–minimum version of the CCDM is also widely studied and used (see for instance [17]). We have checked that the structure functions computed in the two versions of the CCDM do not differ sensibly².

The technique used to compute the physical nucleon state appearing in eq. (1) is based on a double projection of the mean-field solution on linear and angular momentum eigenstates. This technique was already used to compute the static properties of the nucleon [16], the unpolarized and the longitudinally polarized distribution functions [17] and the nucleon electromagnetic form factors [19]. We refer the reader to these references for more details.

The intermediate states labelled by the quantum numbers α in eq. (3) are also computed within the CCDM. Notice that they are admitted in the model, since this has no color. The lightest states contributing to the quark distributions are the diquark states (scalar ud and vectorial uu, ud). These correspond to diagrams in which a quark is extracted and probed by the photon. It turns out that more massive states ($3q1\bar{q}$ states arising from an antiquark insertion) give smaller contributions to the structure functions and are important only at small x . The antiquark distribution receives contributions from the $4q$ states, which correspond to diagrams with a quark insertion. We explicitly found that all these terms saturate with a $\sim 4\%$ accuracy the normalization of the u and d valence distributions. This fulfillment of the valence number sum rule is of course a crucial check of the reliability of our calculation. The momentum sum rule is satisfied as well: in our model [14, 17], at Q_0^2 the valence carries about 75% of the energy-momentum, the remaining part being carried by the mesons and the dielectric field (which, in the spirit of the CCDM, embodies nonperturbative

²The single–minimum CCDM seems to be preferable in the light of quark matter calculations [18].

glue).

The transverse polarization distributions of quarks and antiquarks obtained from eq. (3) using the chiral chromodielectric model are shown in Figs. 1-2. We should recall that the distributions computed in a quark model have no dependence on the momentum transfer. They represent a picture of the nucleon at some low scale Q_0^2 , the “model scale”. Since at such low scales higher twist effects are important, the structure functions obtained in quark models do not necessarily describe the *physical* nucleon at Q_0^2 , but can be used as initial conditions for the Altarelli–Parisi evolution from Q_0^2 to a larger scale, where higher twist contributions are absent. In previous works [14, 17] we showed how to determine the model scale by comparing the model prediction for the valence momentum with the experimental value and found for the CCDM $Q_0^2 = 0.16 \text{ GeV}^2$. We start from this scale the QCD evolution of our transverse polarization densities.

Being chirally odd, $h_1(x, Q^2)$ does not mix with gluon distributions, which are chirally even. Thus its Q^2 evolution at leading order is governed only by the process of gluon emission. The Altarelli–Parisi equation for the QCD evolution of $h_1(x, Q^2)$ is

$$\frac{dh_1^{q,\bar{q}}(x, Q^2)}{d \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P_h(y) h_1^{q,\bar{q}}\left(\frac{x}{y}, Q^2\right), \quad (11)$$

where the leading order splitting function $P_h(y)$ has been computed by Artru and Mekhfi [3] and reads

$$P_h(y) = \frac{4}{3} \left[\frac{2}{(1+y)_+} - 2 + \frac{3}{2} \delta(y-1) \right]. \quad (12)$$

The Mellin transforms of the splitting function $P_h(y)$ are the anomalous dimensions $\gamma_h^{(n)}$ which govern the Q^2 dependence of the moments of h_1 , $h_1^{(n)}(Q^2) \equiv \int_0^1 dx x^{n-1} h_1(x, Q^2)$, according to the multiplicative rule

$$h_1^{(n)}(Q^2) = h_1^{(n)}(Q^2) \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]^{\frac{6\gamma_h^{(n)}}{33-2n_f}}, \quad (13)$$

where n_f is the number of flavors. In particular, since $\gamma_h^{(1)} = -2/3$, the first moment of h_1 and the tensor charge $\delta q \equiv \int dx (h_1^q - h_1^{\bar{q}})$ decrease with Q^2 as

$$\delta q(Q^2) = \delta q(Q_0^2) \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]^{-4/27}. \quad (14)$$

Hence the Q^2 evolution of $h_1^q(x, Q^2)$ and $\delta q(Q^2)$ is different from that of the helicity distributions $\Delta q(x, Q^2)$ and of the singlet axial charge $\Delta\Sigma(Q^2)$. The latter is constant in Q^2 , being related to the matrix element of a conserved current. Therefore, although all existing model calculations (including ours) give results for h_1^q very close to those obtained for Δq , one should keep in mind that this scenario is valid only at the model scale Q_0^2 . At typical experimental scales h_1^q and Δq are different in magnitude and shape as they have evolved differently from similar inputs, and the assumption $h_1^q \simeq \Delta q$ is no longer tenable.

The evolved distribution functions at $Q^2 = 25$ GeV 2 are also shown in Figs. 1–2. The tensor charges at this scale are: $\delta u = 0.969$, $\delta d = -0.250$.

To illustrate the different evolution of the longitudinal and the transverse polarization distributions we compare h_1^u and Δu in Fig. 3. It is evident that, although at Q_0^2 the two distributions are almost identical, after the evolution they are largely different at small x . In particular, the transverse distribution is considerably smaller than the longitudinal one for $x < 0.1$. A similar situation occurs for the d distributions.

Let us turn now to the possible determination of h_1 . The most promising way to detect the transverse polarization distribution is to measure the double-spin asymmetry in the Drell-Yan process with two transversely polarized proton beams. This quantity is given by (see *e.g.* [5]):

$$A_{TT} = a_{TT} \frac{\sum_q e_q^2 h_1^q(x_a, M^2) h_1^{\bar{q}}(x_b, M^2) + (a \leftrightarrow b)}{\sum_q e_q^2 q(x_a, M^2) \bar{q}(x_b, M^2) + (a \leftrightarrow b)}, \quad (15)$$

where we have labeled by a, b the two incoming protons, the virtuality M^2 of the quark and antiquark distributions is the squared mass of the produced dilepton pair, and x_a, x_b, M^2 are related to the center of mass energy \sqrt{s} by $x_a x_b = M^2/s$. The partonic asymmetry a_{TT} is calculable in perturbative QCD [1] and varies between -1 and 1 . The double longitudinal asymmetry A_{LL} has an expression similar to (15), with the transverse distributions replaced by the longitudinal distributions $\Delta q(x, M^2)$.

In Fig. 4 we show our predictions for A_{TT}/a_{TT} at $\sqrt{s} = 100$ GeV 2 and for various M^2 values. For comparison A_{LL}/a_{LL} is also shown. Notice that the transverse asymmetry is an increasing function of the dilepton squared mass; however it remains about one order of magnitude smaller than the longitudinal asymmetry. In Fig. 5 we present A_{TT}

for $x_a - x_b = 0$ as a function of the center of mass energy: one can see that increasing \sqrt{s} leads to a further depletion of A_{TT} . The difference between A_{LL} and A_{TT} is an effect of the different evolution of h_1 and Δq in the small- x region, which dominates the Drell–Yan asymmetries.

The present calculation leads us to conclude that A_{TT} is much smaller than it was expected on the basis of naive estimates. This is confirmed by a model-independent study of Drell–Yan asymmetries which will be reported in a separate paper. The extraction of h_1 is then a major challenge for experimentalists but is certainly worth attempting as it can add an important piece of information to our knowledge of the proton.

One of us (VB) would like to thank the Institute of Nuclear Theory at the University of Washington for its hospitality and the U.S. Department of Energy for partial support during the completion of this work.

References

- [1] J.P. Ralston and D.E. Soper, Nucl. Phys. B152 (1979) 109.
- [2] R.L. Jaffe and X. Ji, Phys. Rev. Lett. 67 (1991) 552; Nucl. Phys. B375 (1992) 527.
- [3] X. Artru and M. Mekhfi, Z. Phys. C45 (1990) 669.
- [4] J.L. Cortes, B. Pire and J.P. Ralston, Z. Phys. C55 (1992) 409.
- [5] X. Ji, Phys. Lett. B284 (1992) 137.
- [6] R.L. Jaffe and X. Ji, Phys. Rev. Lett. 71 (1993) 2547.
- [7] H. He and X. Ji, Phys. Rev. D52 (1995) 2960.
- [8] A. Yokosawa, in Proc. 11th Int. Symp. on High Energy Spin Physics (Bloomington, IN, 1994), AIP Press, 1995.
- [9] C. Bourrely and J. Soffer, Nucl. Phys. B423 (1994) 329.
- [10] J. Soffer, Phys. Rev. Lett. 74 (1995) 1292.
- [11] G.R. Goldstein, R.L. Jaffe and X. Ji, Phys. Rev. D52 (1995) 5006.
- [12] H.J. Pirner, Prog. Part. Nucl. Phys. 29 (1992) 33.
M.C. Birse, Prog. Part. Nucl. Phys. 25 (1990) 1.
M.K. Banerjee, Prog. Part. Nucl. Phys. 31 (1993) 77.
- [13] A.W. Schreiber, A.I. Signal and A.W. Thomas, Phys. Rev. D44 (1991) 2653.
- [14] V. Barone and A. Drago, Nucl. Phys. A552 (1993) 479; *ibid.* A560 (1993) 1076 (E).
- [15] V. Barone, paper in preparation.
- [16] T. Neuber, M. Fiolhais, K. Goeke and J.N. Urbano, Nucl. Phys. A560 (1993) 909.
- [17] V. Barone, A. Drago and M. Fiolhais, Phys. Lett. B338 (1994) 433.
- [18] A. Drago, M. Fiolhais and U. Tambini, Nucl. Phys. A588 (1995) 801.
A. Drago, M. Hjorth-Jensen and U. Tambini, Phys. Lett. B, in press.
- [19] A. Drago, M. Fiolhais and U. Tambini, submitted for publication.

Figure Captions

Fig. 1 The transverse polarization distribution of quarks $h_1(x)$ at the model scale $Q_0^2 = 0.16 \text{ GeV}^2$ (dashed line: h_1^u ; dotted line: h_1^d) and at $Q^2 = 25 \text{ GeV}^2$ (solid line: h_1^u ; dot-dashed line: h_1^d).

Fig. 2 Same as Fig. 1 for the antiquark distributions $h_1^{\bar{q}}$.

Fig. 3 Comparison of the evolution of the transverse polarization distribution h_1^u (dashed line: $Q^2 = Q_0^2 = 0.16 \text{ GeV}^2$; solid line: $Q^2 = 25 \text{ GeV}^2$) and of the longitudinal polarization distribution Δu (dotted line: $Q^2 = Q_0^2 = 0.16 \text{ GeV}^2$; dot-dashed line: $Q^2 = 25 \text{ GeV}^2$).

Fig. 4 Predictions for the Drell-Yan double transverse asymmetry A_{TT}/a_{TT} (dot-dashed line: $M^2 = 50 \text{ GeV}^2$; dashed line: $M^2 = 25 \text{ GeV}^2$; solid line: $M^2 = 10 \text{ GeV}^2$). For comparison, the double longitudinal asymmetry A_{LL}/a_{LL} is shown for $M^2 = 10 \text{ GeV}^2$ (dotted line). All curves are obtained with $\sqrt{s} = 100 \text{ GeV}$.

Fig. 5 Dependence on M^2 of the transverse double spin asymmetry at $x_a - x_b = 0$ (dot-dashed line: $\sqrt{s} = 100 \text{ GeV}$; dashed line: $\sqrt{s} = 300 \text{ GeV}$; solid line: $\sqrt{s} = 500 \text{ GeV}$).









